CLUSTERING MIXTURE MODELS :

BEYOND GAUSSIANS

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Central Open Question

Can we efficiently cluster mixtures of non-spherical log-concave distributions ?

Plan :

- Problem Setup
- Bottknecks

be the uniform distribution over a convex body Let \bigcup $K \in \mathbb{R}^d$ with mean μ and Covariance Σ . (log-concave)

Ex. 1 Unit sphere 2 Unit Cube 3 Simplex

Let $M = \sum w_i D(p_i, \Sigma_i)$ be a mixture of log-concave et 19 = 2 a
ie [E]
distributions.

Let
$$
M = \sum_{i \in [k]} w_i D(p_i, \Sigma_i)
$$
 be a mixture of log-concave

distributions.

Definition: M is "clusterable" if
$$
\forall i, j \in \Sigma
$$

\n $D(\mu_i, \Sigma_i)$ and $D(\mu_j, \Sigma_j)$ are either

\n① Mean- separated

\n② Specifically - separated

\n③ Shell/Frobenius-sepeated

Definition : Mis "clusterable" if \forall ij E IR] $D(\mu_i, \overline{z}_i)$ and $D(\mu_j, \overline{z}_j)$ are either Q Mean - Separated ^②Spectrally separated ③ Shell/Frobenius - separated

- For Gaussians , this captures separation in total-varition distance [B-Kothari'20, Diakonikolas - Hopkins-Kane-Karmalkar '20]
- Implies all known^{*} notions of parameter seperation
- Generalizes clustering spherical mixtures

Definition : Mis "clusterable" if \forall ij E IR] $D(\mu_i, \overline{z}_i)$ and $D(\mu_j, \overline{z}_j)$ are either

^O Mean-separated : Fr ^s . t .
1. perated: $\exists v$ s.t.
 $\langle \psi_i-\psi_j, v \rangle \gg v^7 (z_i + \overline{z}_j)_{\vee}$

Definition : M is "clusterable" if H i,j \in Ω k] $D(\mu_i, \overline{z}_i)$ and $D(\mu_j, \overline{z}_j)$ are either

① Mean- separated:
$$
\exists v \text{ s.t.}
$$

\n
$$
\langle v_{i} - v_{j,v} \rangle \gg v^{7} (z_{i} + z_{j})_{v}
$$
\n② Spectrally- separated: $\exists v \text{ s.t.}$

\n
$$
v^{T} \Sigma_{i} v \gg v^{T} \Sigma_{2} v.
$$

Definition : M is "clusterable" if H i,j \in Ω k] $D(\mu_i, \overline{z}_i)$ and $D(\mu_j, \overline{z}_j)$ are either

^O Mean-separated : Fr ^s . t . [Ni-P, ^v .vi(zi ⁺ ^z ;) v . ② spectrally separated : In s. t . Ziv TI, ^v IzV . ③ Shell/Frobenius separated : 1) ZI-IIEIIzzy p Ij

Recap : Gaussian Mixtures

^⑦ Hyper-contractivity of linear forms => Mean-Separated Clusting [Kothari-Steinhardt ¹⁸, Hopkins - ^L: 18]

② Hyper-contractivity of deg-2 polynomials => Shell/Frobenius-Sep Clustering [B.-Kothavi'20, Diakonikolaz-Hopkins-Kare-Karmallzar'20] Anti-Concentration => Spectral-Sep Clustering

[B.-Kothavi'20, Diakonikolaz-Hopkins-Kanc-Karmallzar'20]

Recap : Gaussian Mixtures

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[B.-Kothavi'20, Diakonikolaz-Hopkins-Kanc-Karmallzar'20]

* SoS certifiable versions of these conditions imply efficient algorithms

Beyond Gaussians : Claim : Any log-concave distribution satisfies : ^⑦ Hyper-contractivity of linear forms ② Hyper-contractivity of deg-2 polynomials ③ Anti-concentration

Beyond Gaussians : Claim : Any log-concave distribution satisfies : ^⑦ Hyper-contractivity of linear forms ② Hyper-contractivity of deg-2 polynomials ③ Anti-concentration Proof Sketch: [↓] veIRd , the PDF is point-wise bounded by exp) <x, v >c) . How do we get efficient algorithms?

Analytic Certificates

Def [Certifiable Hycontractivity of linear forms]:

$$
\forall v \in \mathbb{R}^{d}, C_{k} \cdot (E \le x, v^{2})^{k} - E \le x, v^{2k} = Sos(v).
$$

Det [Certifiable Hycontractivity of linear forms]: $\forall v \in \mathbb{R}^d, \qquad C_k \cdot (\underbrace{E \prec x, v^2}_{variance})^k - \underbrace{E \prec x, v^2}_{lump} = \text{Sos}(v).$ linear form

4 apriori, a statement with infinite constraints

La admits a polynomial size representation

Analytic Certificates imply Efficient Algorithms

Mean Separation

Shell separation Clustering

Spectral Separation

Analytic Certificates imply Efficient Algorithms

Certifiable Hypercontractivity > Mean Separation of linear forms & Certifiable Hypercontractivity of deg ² polynomials > shell separation Clustering Certifiable Anti-concentration/Spectral Separation

Analytic Certificates imply Efficient Algorithms

Certifiable Hypercontractivity < Mean Seperation Poincare Tri-stained of linear forms Inequality --- Certifiable Hypercontractivity Prog of deg ² polynomials > Shell separation Clustering Certifiable Anti-concentration/Spectral Separation

Analytic Certificates imply Efficient Algorithms KLS Conjecture Certifiable Hypercontractivity < Mean Seperation [~] Tristent of linear forms Poincare Inequality --- Certifiable Hypercontractivity & Prog of deg ² polynomials > Shell separation Clustering Certifiable Anti-concentration/Spectral Separation

Analytic Certificates imply Efficient Algorithms KLS Conjecture - Certifiable Hypercontractivity < Mean Seperation Poincare Kothari-Steinhardt of linear forms & Inequality --- Certifiable Hypercontractivity Prog of deg ² polynomials ⁷ shell separation Clustering Rotational > Invariance Certifiable Anti-concentration / Spectral Separation Raghavendra-You Karmalkar-Klivans-Kothari B. -Kothari

Analytic Certificates imply Efficient Algorithms KLS Conjecture Certifiable Hypercontractivity < Mean Seperation [~] Tristent of linear forms Inequality Certifiable Hypercontractivity Poincare 2000s Prog of deg ² polynomials > Shell separation& Clustering Relaxing This Rotational Talk > Certifiable Anti-concentration / Spectral Separation Invariance ⁷ List-Decodable Regression

Certifiable Anti-Concentration : Beyond Gaussians joint w) Provesh Kothari, Goutham Rajendran, Madhur Tulsiani & Aravindan Vijayaraghavan

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Letifiable Anti-Concentrelation: Beyond Gaussians
\njoint u) Proveh. Kethani, Goulham Rajendan, Madhar Tulsiani & Anuindan Vijayvaghavar
\nDef
$$
\bigcup
$$
 flati-Concentretriou]: Given a distri batch. Dorer \mathbb{R}^d , \forall directions \vee
\nand all intervals \mathbb{I} of length $8\sqrt{v^T\Sigma v}$, \forall
\n \mathbb{R}^d , \forall directions \mathbb{R}^d

\nThen \mathbb{D} is 8 -anti-concentrated.

\nThen \mathbb{D} is 8 -anti-concentrated.

Certifiable Anti-Concentration : Beyond Gaussians $Def[\bigtriangleup]$ Anti-concertration]: Given n iid samples $\{x_i\}_{i\in\mathbb{N}_J}$ from a distribution D , for all directions r in \mathbb{R}^d , it $Pr \left[\begin{array}{cc} \langle x_i, v \rangle & \leq \delta \cdot v^T \Sigma v \end{array} \right] \leq \delta$ rections r in \mathbb{R}^d , if
 \mathbb{R}^d \mathbb{R}^d , \mathbb{R}^d
 \mathbb{R}^d \mathbb{R}^d \mathbb{R}^d , \mathbb{R}^d \mathbb{R}^d & Certificable Anti-Concentration: Beyond Gouesians

Def [Anti-concentration]: Given n iid samples {xi}:_{EENI} from a distribution D

for all directions r in \mathbb{R}^d , if
 P_{rr} [$\langle x_i, v \rangle \stackrel{?}{\leq} \frac{1}{8} \cdot v^T \Sigma r$] $\frac{1}{s}$ $\frac{1}{s}$ $\frac{1}{s}$ $\frac{1}{s}$ Can we formulate this as an integer program ?

Certifrable Anti-Concentration: Integer Program Given $\{x_i\}_{i\in[n]}$, \mathcal{D}_{ξ} \mathcal{D}_{ξ} and \mathcal{D}_{ξ} are the contraction \mathcal{D}_{ξ} . Criter samples \mathcal{D}_{ξ} is \mathcal{D}_{ξ} from a distribution \mathcal{D}_{ξ} \overline{D} is S -anti-concertrated if for all directions r in \mathbb{R}^d , $P_x \left[\left| \langle x, v \rangle \right| \leq \left| \hat{s} \right| \sqrt[n]{\sum x} \right] \leq \hat{s}$

$$
\max_{v \in \mathbb{R}^d} \quad \frac{1}{n} \sum_{i \in \text{Ln}J} \mathbb{1} \left[\begin{array}{c} \langle x_i, v \rangle^{\epsilon} \leq \delta v^T \Sigma v \\ \end{array} \right]
$$

Certifrable Anti-Concentration: Integer Prognam Given $\{x_i\}_{i\in[n]}$, Det [Arti-concentration]: Given samples $\{x_i\}_{i\in\mathbb{C}\cup\mathbb{T}}$ from a distribution D , D is S-anti-concentrated if for all directions r in R^d, $P_Y \left[| \langle x_i, v \rangle | \leq \delta v^T \Sigma v \right] \leq \delta$

$$
\max_{v \in \mathbb{R}^d} \quad \frac{1}{n} \sum_{i \in \mathbb{Z}^{n}} \mathbb{1} \left[\begin{array}{c} \langle x_i, v \rangle^2 \leq \sum v^T \sum v \end{array} \right]
$$

If OPT \leqslant S , then the uniform distribution over $\{x_i\}_{i\in [n]}$ is Claim: S-anticoncentrated.

Certifiable Anti-Concentration : Integer Program

$$
Given \quad \{x_i\}_{i\in\mathbb{C} n} \quad
$$

 $Def[\Sigma]$ Anti-concentration]: Given samples $[x_i]_{i\in\mathbb{N}]}$ from a distribution D , D is S -anti-concentrated if for all directions r in \mathbb{R}^d , $Pr[\exists x: y \forall x \in S \cdot \sqrt{X}] \leq S$

$$
\begin{array}{ll}\n\text{max} & \frac{1}{n} \sum_{i \in \text{Ln} 1} w_i, & \text{s.t.} \\
\forall, \omega & \text{in } \text{len} 1 & \omega_i = w_i \\
\text{the } w_i \in \text{Ln} 1 & \omega_i & \text{[Indicate values]} \\
\text{the } w_i \leq x_i, v \leq \omega_i \leq v \leq \sum_{i} \text{[carding connected points]}\n\end{array}
$$

Certifiable Anti-Concentration : Integer Program

$$
Given \quad \{x_i\}_{i\in\mathbb{C} n} \quad \ \,
$$

 $Def[\Sigma]$ Anti-concentration]: Given samples $[x_i]_{i\in\mathbb{N}_1}$ from a distribution D , \overline{D} is δ -anti-concertrated if for all directions r in \mathbb{R}^d , $Pr[\vert \langle x_i, v \rangle] \leq \delta v^T \Sigma v \int \leq \delta$

$$
\begin{array}{ll}\n\text{max} & \frac{1}{n} \sum_{i \in \text{En1}} w_i, & \text{s.t.} \\
\forall i \in \text{In1} & w_i^2 = w_i \quad [\text{Indicate variables}] \\
\text{HieEn1} & w_i < x_i, v \le w_i \le v_i \le v_i \text{Counting concentrated points} \\
\text{How do we activity upper bounds on the objective value of this program}\n\end{array}
$$

Certifiable Anti-Concentration : EfficientCertificates

$$
\mathcal{A} := \begin{cases} \n\begin{array}{ccc}\n\mathcal{H} & \text{if } \mathcal{L} & \mathcal{L} \\
\mathcal{L} & \mathcal{L} & \mathcal{L} \\
\mathcal{L} & \mathcal{H} & \mathcal{L} \\
\mathcal{H} & \mathcal{L} & \mathcal{L} \\
\mathcal{H} & \mathcal{L} & \mathcal{L}\n\end{array}\n\end{cases}
$$

Key Idea : Derive an upper bound on the objective in the sum-of-squares proof system

=> Efficiently representable certificate

Centifiable Anti-Concentroduction:
$$
\exists
$$
 fticit Certificates

\nKey Tdea: Derive an upper bound on the objective

\nin the sum-of-spuares proof system

\n \Rightarrow \exists fticent\n \Rightarrow \exists fticient\n \Rightarrow \exists fticient\n \Rightarrow \Rightarrow \exists fticient\n \Rightarrow $$

Certif table: Anti–Concentration:
$$
E_{\text{eff}}
$$
 *c*ert (crit) *cc* E_{eff} $A := \begin{cases} \frac{4}{16} \text{cm} & w_c^2 = w_c \\ \frac{4}{16} \text{cm} & w_c < x_c, v > c \leq 8. v^T \Sigma_v \end{cases}$

$$
Main Theorem: Given n samples from a 'reasnably anti-concentrated' distribut-thene is a degree- O_{δ} (log d) certificate of anti-concentration, ie.
 $A \vdash \delta = \bot \sum_{n \text{ terms}} \omega_{i} \geq O_{\delta}$
 O_{δ}
$$

Certifiable Anti-Concentration : EfficientCertificates fitEn] we ⁼ Wi A : ⁼ ^S fit[n] wi(xi , v ^S . vZv J MainTheorem : Given n samples from ^a "reasonably anti-concentrated" distribution there is a degree-Ologd) certificate of anti-concentration , i . e . ^A + (S - ⁺ nie[n] ^z wi o Ollogd) ^① Running Time : nOsblogd((Casi-polynomial (② S - Dependence : exp(/(*)"s) (Doubly exponential) ③ No direct sum-of-squares proof

Centificable: Anti-concentration:
$$
\frac{1}{2} \text{th circuit Centificates}
$$

\nHere $\text{at } \mu_{i} = \int_{\mu_{i} \in \mathbb{N}} \mu_{i} \cdot \mu_{i} \cdot \mu_{i} \cdot \mu_{i} \cdot \mu_{i}$

\nHere $\mu_{i} = \int_{\mu_{i} \in \mathbb{N}} \mu_{i} \cdot \mu_{i} \cdot \mu_{i} \cdot \mu_{i} \cdot \mu_{i}$

$$
Main Theorem
$$
; Given n samples from a 'reasonably anti-concentated' distribut-
then is a degree- O_8 (log d) certificate of anti-concentretion, i.e.
 $A \vdash_{S} S - \bot \sum_{h \text{ isfin } 3} \omega_i \geq O_3$.

$$
A \vdash \S - \bot \sum_{\substack{n \text{ is a } n}} w_i \geq 0 \S .
$$

^⑦ Certificate works for affine transformations of uniform distributions over Lp balls, anti-concentrated product distributions etc. anti-concentrated product distributions etc.
(5) All prior certificates required rotational invariance.

Certifiable Anti-Concentration : EfficientCertificates

MainTheorem : Given n samples from ^a "reasonably anti-concentrated" distribution there is a degree-Ologd) certificate of anti-concentration , i . e .

Applications :

1. Clustering : A
$$
n^{\circ}C^{\circ}d
$$
 time *lrobust* > algorithm for clustering
\nspectra l_{\circ} -separated components.
\n $\rightarrow A n^{\circ}L \cdot E^{\circ}d$ times (robust) algorithm for clustering mixtures.

Certifiable Anti-Concentration : EfficientCertificates

MainTheorem : Given n samples from ^a "reasonably anti-concentrated" distribution there is a degree-Ologd) certificate of anti-concentration , i . e .

Applications :

\n- 1. Clustering : A
$$
n^{\circ}S^{\text{log }d}
$$
 time. Involust) algorithm for clustering separately -separated. Components.
\n- 2. List-Decodable Regression: A $n^{\circ}C_{\text{max}}$ (robust) algorithm for clustering mixtures.
\n- 2. List-Decodable Regression: A $n^{\circ}C_{\text{max}}(log d)$ time algorithm for our polymorphism of the set of size $O^{\text{log }d}$ as follows:
\n

Certifiable Anti-Concentration : Overview

Starting Point : For uniform distributions over Lp balls, the marginals along random directions are Gaussian-like .

Certifiable Anti-Concentration: Overview Starting Point : For uniform distributions over Lp balls, the marginals along random directions are Gaussian-like .

Lemma: $H = ||v||_{v}^{4} \le \lambda \cdot ||v||_{2}^{4}$ 3 analytic density

Certifiable Anti-Concentration: Overview Starting Point : For uniform distributions over Lp balls, the marginals along random directions are Gaussian-like .

Lemma: $H = ||v||_{v}^{4} \le \lambda \cdot ||v||_{2}^{4}$ 3 analytic density

- > We provide an explicit sum-of-squares proof of this In the indeterminate v).

Certifrable Anti-Concentration: Overview Starting Point: For uniform distributions over Lp balls, the marginals along random directions are Gaussian-like.

$$
L_{emm2}: \quad \n\downarrow \quad \left\| v \right\|_{v}^{\frac{1}{4}} \leq \lambda \cdot \left\| v \right\|_{2}^{\frac{1}{4}}
$$
\n
$$
= \frac{\left(2k\right)!}{2^{k}k!} \quad \text{if} \quad \lambda \cdot k^{\frac{k}{2}} \quad \text{if} \quad \left\| v \right\|_{2}^{\frac{2k}{2}} \leq \lambda \cdot k^{\frac{2k}{2}} \quad \text{if} \quad \left\| v \right\|_{2}^{\frac{2k}{2}} \leq \lambda \cdot k^{\frac{k}{2}} \quad \text{if} \quad \left\| v \right\|_{2}^{\frac{2k}{2}} \leq \lambda \cdot k^{\frac{2k}{2}} \quad \text{if} \quad \left\| v \right\|_{2}^{\frac{2k}{2}} \leq \lambda \cdot k^{\frac{2k}{2}} \quad \text{if} \quad \left\| v \right\|_{2}^{\frac{2k}{2}} \leq \lambda \cdot k^{\frac{2k}{2}} \quad \text{if} \quad \left\| v \right\|_{2}^{\frac{2k}{2}} \leq \lambda \cdot k^{\frac{2k}{2}} \quad \text{if} \quad \left\| v \right\|_{2}^{\frac{2k}{2}} \leq \lambda \cdot k^{\frac{2k}{2}} \quad \text{if} \quad \left\| v \right\|_{2}^{\frac{2k}{2}} \leq \lambda \cdot k^{\frac{2k}{2}} \quad \text{if} \quad \left\| v \right\|_{2}^{\frac{2k}{2}} \leq \lambda \cdot k^{\frac{2k}{2}} \quad \text{if} \quad \left\| v \right\|_{2}^{\frac{2k}{2}} \leq \lambda \cdot k^{\frac{2k}{2}} \quad \text{if} \quad \left\| v \right\|_{2}^{\frac{2k}{2}} \leq \lambda \cdot k^{\frac{2k}{2}} \quad \text{if} \quad \left\| v \right\|_{2}^{\frac{2k}{2}} \leq \lambda \cdot k^{\frac{2k}{2}} \quad \text{if} \quad \left\| v \right\|_{2}^{\frac{2k}{2}} \leq \lambda \cdot k^{\frac{2k}{2}} \quad \text{if} \quad \left\| v \right\|_{2}^{\frac{2k}{2}} \leq \lambda \cdot k^{\
$$

How do we handle the remaining directions?

$$
15.36 \text{ not admit a small cover } \ell_{\mathsf{X}}: V = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2\sqrt{d}} \cdots \frac{1}{2\sqrt{d}}\right)
$$

Certifiable Anti-Concentration: Overview

Lemma: If
$$
||v||_4^4 \le \lambda \cdot ||v||_2^4
$$

\nthen $||E \le x, v \le 2k = \frac{(2k)!}{2^{k}k!} ||v||_2^2 + \lambda \cdot k^{k} ||v||_2^{2k}$
\n $lim_{x \to D} 4 \le x \le 2^{k} \le 2^{k$

$$
\text{If } \forall_H \text{ } \text{If } \text{ } \forall H \text{ } \text{If } \text{ } \text{
$$

Does not seem to help if v is an indeterminate.

Certifiable Anti-Concentration: Overview

① Switch to dual :

Lemma: Any direction v can be decomposed in
$$
v_1 + v_1
$$
 s.t. v_1 has
\n y_1 non-zero coordinates and v_1 is analyticably dense i.e.
\n
$$
\|v_1\|_1^k \le \lambda \|v_1\|_2^4
$$

But the value:

\n
$$
\frac{1}{2} \int \frac{1}{x} \, dx + \frac{1}{2} \int \frac{1}{x} \, dx
$$
\nLet: $\frac{1}{2} \int \frac{1}{x} \, dx$ and $\frac{1}{2} \int \frac{1}{x} \, dx$ and $\frac{1}{2} \int \frac{1}{x} \, dx$ are $\frac{1}{2} \int \frac{1}{x} \, dx$ and $\frac{1}{2} \int \frac{1}{x} \, dx$ are $\frac{1}{2} \int \frac{1}{x} \, dx$ and $\frac{1}{2} \int \frac{1}{x} \, dx$ are $\frac{1}{2} \int \frac{1}{x} \, dx$ and $\frac{1}{2} \int \frac{1}{x} \, dx$ are $\frac{1}{2} \int \frac{1}{x} \, dx$ and $\frac{1}{2} \int \frac{1}{x} \, dx$ are $\frac{1}{2} \int \frac{1}{x} \, dx$ and $\frac{1}{2} \int \frac{1}{x} \, dx$ are $\frac{1}{2} \int \frac{1}{x} \, dx$ and $\frac{1}{2} \int \frac{1}{x} \, dx$ are $\frac{1}{2} \int \frac{1}{x} \, dx$ and $\frac{1}{2} \int \frac{1}{x} \, dx$ are $\frac{1}{2} \int \frac{1}{x} \, dx$ and $\frac{1}{2} \int \frac{1}{x} \, dx$ are $\frac{1}{2} \int \frac{1}{x} \, dx$ and $\frac{1}{2} \int \frac{1}{x} \, dx$ are $\frac{1}{2} \int \frac{1}{x} \, dx$

Certifiable Anti-Concentration : Overview Lemma : Any direction v can be decomposed in $v_{\rm L}$ + $v_{\rm H}$ \cdot 3. t . $v_{\rm L}$ has y_λ non-zero coordinates and y_μ is analytically dunse i.e. ① Switch to dual : $\left\| \left\langle \mathbf{v}_{\mathsf{H}}\right\rangle \right\| _{\mathsf{V}}^{\mathsf{V}}\leq \left\| \mathbf{v}_{\mathsf{H}}\right\| _{\mathsf{V}}^{\mathsf{H}}$. F_{act} : if H degt pseudo-distributions μ \mathbb{E}_{ν}^{\sim} p(x) \approx , $\frac{1}{2}$ a sos proof of ρ (x); o.

a ② Assume for contradiction there is such pseudo-distribution

If ③ Condition on "large" coordinates of r by re-weighting the pando-distribution

Certifiable Anti-Concentration : Overview Lemma : Any direction v can be decomposed in $v_{\rm L}$ + $v_{\rm H}$ \cdot 3. t . $v_{\rm L}$ has y_λ non-zero coordinates and y_μ is analytically dunse i.e. ① Switch to dual : $\left\| \left\langle \mathbf{v}_{\mathsf{H}}\right\rangle \right\| _{\mathsf{V}}^{\mathsf{V}}\leq \left\| \mathbf{v}_{\mathsf{H}}\right\| _{\mathsf{V}}^{\mathsf{H}}$. F_{act} : if H degt pseudo-distributions μ \mathbb{E}_{ν}^{\sim} p(x) \approx , $\frac{1}{2}$ a sos proof of ρ (x); o. ② Assume for contradiction there is no such pseudo-distribution

③ Condition on "large" coordinates of r by "re-weighting the pando-distribution ^① The resulting rector is analytically densa and we can invoke the explicitproof for analytically dense directions

Centifiable Anti-Concentration: Toy Application.

\n
$$
\{x_{i}\}_{i \in Inj}
$$
\nsamples from M = $\frac{1}{2}D(0, \overline{2},) + \frac{1}{2}D(0, \Sigma_{2})$ s.f.

\n
$$
x_{i} + y_{i} = \sqrt{2}, y \in \sqrt{2}, y_{i} \in \
$$

$$
\mathcal{A} := \begin{cases} \n\forall i \in [n] & \omega_{c} = \omega_{i} \\
\forall i \in [n] & \omega_{i} \leq x_{i}, v \leq \epsilon \omega_{i} \leq \frac{2}{\pi} \sqrt{n} \\
\hline\n\sum \omega_{i} = \sqrt{2}\n\end{cases}
$$

Centifiable: Anti-Concentration: Toy Application

\n
$$
\{x_{i}\}_{i \in \mathbb{Z}^{n}} \quad \text{samples from } M = \frac{1}{2} D(0, \Sigma_{1}) + \frac{1}{2} D(0, \Sigma_{2}) \text{ s.t.}
$$
\n
$$
x_{1} \quad \forall v \quad \overline{v}^{\top} \Sigma_{i} v \leq v^{\top} \Sigma_{i} v
$$
\n
$$
x_{2} \quad \overline{v}^{\top} \Sigma_{i} v \leq v^{\top} \Sigma_{i} v \leq v^{\top} \Sigma_{i} v
$$
\nThus, assume M is isotopic.

$$
A := \begin{cases} \forall i \in [n] & \omega_{i} \in \omega_{i} \\ \forall i \in [n] & \omega_{i} \geq x_{i}, v \geq \epsilon \omega_{i} \leq ||v||^{2} \\ \sum \omega_{i} = \gamma/2 \end{cases}
$$

Algorithm :

\n- 1. Compute a
$$
dy = 0
$$
 [log d) pseudo-distribution
\n- 2. Sample $g \sim N(0, \vec{E} \cdot v^T)$
\n- 3. Output $g / \log |I|$
\n

Verifyable: Anti-Concentration: Toy Application

\n
$$
\{x_{i}\}_{i\in Inj}
$$
\nsamples from M = $\frac{1}{2}D(0,\overline{z}_{i})+\frac{1}{2}D(0,\overline{z}_{i}) \text{ s.t.}$

\n
$$
x_{i} = \begin{cases}\n\text{WieInj} & \text{with } \omega_{i} = \omega_{i} \\
\frac{1}{2}D(0,\overline{z}_{i})+\frac{1}{2}D(0,\overline{z}_{i}) & \text{with } \omega_{i} = \omega_{i} \\
\frac{1}{2}D(0,\
$$

Key Soslemma:
\n
$$
A + \{\langle \Sigma, vV \rangle \le O(\xi) |vV|^4 \}
$$

\n $\{\sum_{i} \check{E} vT \} \le \tilde{E} \langle \Sigma, vV \rangle \le O(\xi)$
\n $\langle \Sigma, \check{E} vT \rangle \le \tilde{E} \langle \Sigma, vV \rangle \le O(\xi)$
\nObserve $\tilde{E} g g^{-1} = \tilde{E} vV^{-}$ and the claim follows from Markov's

Take aways : KLS Conjecture Certifiable Hypercontractivity < Mean Seperation [~] Tristent of linear forms Poincare 2000s Inequality Certifiable Hypercontractivity & Prog of deg ² polynomials > Shell separation Clustering Uniform over This ↳ balls Talk > Certifiable Anti-concentration / Spectral Separation ⁷ List-Decodable Regression